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Finite-element analysis of setting expansion in dental gypsum-bonded investment

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A parameter study with use of the finite-element method (FEM) was conducted for the determination of the setting expansion of gypsum-bonded investment. In this study the FEM was applied to the setting expansion within an elastic ring, and the results of FEM analysis were compared with the values measured and the values analytically obtained. The agreement of the results was clarified when the step number for calculation was increased. The present formulae appear to be reasonable for comparison of the value for setting expansion.

1. Introduction

In dental metal casting, gypsum-bonded investment as a mould material requires setting expansion to compensate for thermal shrinkage of the molten metal [1]. Because both the pattern invested in a gypsum-bonded investment and the casting ring surrounding the mould restrict the setting expansion, the setting expansion is not uniform within the casting ring. Therefore, the investment could deform, affecting the casting accuracy of the metal crown [1]. We have already introduced a method of calculating the setting expansion of gypsum-bonded investment for numerical analysis of the deformation of an investment, and have applied this method to setting expansions under a simple condition [2, 3]. The purpose of this study was to apply the finite-element method (FEM) to determine the setting expansion of an investment for the further investigation of setting expansion under more practical conditions. It is thus clarified whether the calculated values of the setting expansion within an elastic ring by the FEM are in good agreement with the results previously reported [3].

2. Method

2.1. FEM for expansion of an investment

As has been reported [1], the setting expansion of an investment under loading conditions can be calculated by using elastic analysis, because the relationships between stress and strain for an investment become the same as for an elastic body when $E'/ka_0 \exp(-kt)$ is substituted by E, $\partial\varepsilon/\partial t$ by ε, ν' by ν and $ka_0 \exp(-kt)$ by αT (see Appendix A).

For the FEM, first, time is divided into SN steps (see Appendix B).

The relationships between the total step number SN and time duration TD(N) in step N are

$$\mathrm{TD}(1) = t_{\mathrm{END}}(\mathrm{RA} - 1)/(\mathrm{RA}\hat{}\,\mathrm{SN} - 1) \quad (1)$$

$$\mathrm{TD}(I) = [\mathrm{RA}\hat{}\,(I - 1)] * \mathrm{TD}(1) \quad (2)$$

$$\mathrm{TIM}(N) = \sum_{K=1}^{N} \mathrm{TD}(K) \quad (3)$$

where RA is a division ratio, and $t_{\mathrm{END}} = 120$ min.

Consider expansion during the finite time duration Δt in time step N. The relationship between stress and strain (see Equation A1), for example, becomes

$$\Delta\varepsilon = ka_0 \exp(-kt) \times (1 + \sigma_{xx}/E' - \nu' \sigma_{yy}/E' - \nu' \sigma_{zz}/E')\Delta t \quad (4)$$

where $E' = 5.0\ \mathrm{kg\,cm^{-2}}$, $\nu' = 0.2$, $k = 0.032\ \mathrm{min^{-1}}$ and $a_0 = 0.009$.

In this step N, the following values are used for the FEM:

$$\mathrm{EI}(N) = E'/[ka_0 \exp\{-k[\mathrm{TIM}(N) - \mathrm{TD}(N)/2]\}\mathrm{TD}(N)]$$

for $E'/[ka_0 \exp(-kt)\Delta t]$ and

$$\mathrm{TC}(N) = ka_0 \exp\{-k[\mathrm{TIM}(N) - \mathrm{TD}(N)/2]\}\mathrm{TD}(N)$$

for $ka_0 \exp(-kt)\Delta t$

in Equation 4. Then Equation 4 becomes

$$\Delta\varepsilon = [\sigma_{xx} - \nu'(\sigma_{yy} + \sigma_{zz})]/\mathrm{EI}(N) + \mathrm{TC}(N) \quad (5)$$

where TC(N) is a free setting expansion in step N.

Equation 5 indicates that the deformation of the investment in time step N is identical to the deformation of an elastic body with Young's modulus EI(N), Poisson's ratio 0.2 and uniform thermal expansion of $\alpha T = \mathrm{TC}(N)$. Therefore, FEM analysis for an elastic body is available for the calculation of the setting expansion when the boundary conditions are given [4].

In Fig. 1 the relationships between N and TC(N) at ratios of RA = 1.0, 1.2, 1.4 and 1.6 are shown. In this

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study RA was determined as the deviation of TC(N) remains small. Therefore, RA = 1.4 at SN = 10 and RA = 1.2 at SN = 20.

2.2. Application to setting expansion within an elastic ring

The FEM was applied to the setting expansion of an investment within an elastic ring under the same conditions as in [3]. The elastic rings had an inner radius of 22 mm, height of 10 mm, and three different outer radii of 22.3, 23.0 and 25.0 mm. Fig. 2 shows the finite-element model: the investment was divided into 48 triangular elements and the ring into eight elements. The Young's modulus and Poisson's ratio of the ring were 30 000 kg cm^{-2} and 0.33, respectively.

Fig. 3 shows the flow chart of analysis and boundary conditions. Consider the calculation in step N.

The program can be divided into two parts (I and II) as follows.

I. In the former part of this step N, the amount of deformation of the investment is calculated. As boundary conditions, the displacement at $r = 0$ in the radial direction is considered to be zero and the displacement in the z-axis direction at nodal point 15, the half-height of the ring, is also zero. Furthermore, the force acts on the investment mould from the ring at contact points between the investment and the ring, and the value of this external force has to be taken in as a boundary condition, as will be seen below. Under these boundary conditions the displacement of nodal point I of the investment in step N, TXI(N, I) and TYI(N, I), can be calculated as the deformation of an elastic body with Young's modulus of E = EI(N), Poisson's ratio of 0.2 and uniform thermal expansion of αT = TC(N) by the FEM for an elastic body. Finally, the displacement of nodal point I of an investment at TIM(N) becomes $\mathrm{DXI}(N, I) = \sum_{K=1}^{N} \mathrm{TXI}(K, I)$ and $\mathrm{DYI}(N, I) = \sum_{K=1}^{N} \mathrm{TYI}(K, I)$.

IIa. In the latter part of this step N, the displacements of the ring at nodal point K, TXE(N, K) and TYE(N, K), are calculated by FEM analysis, and the boundary conditions are as follows: as shown in Fig. 3, the number of contact points between the investment and elastic body, NIE, is 5, and nodal points 7, 14, 21, 28 and 35 of the investment contact with nodal points 1, 3, 5, 7 and 9 of the ring, respectively. At these contact points between the investment and the ring, the displacement of the investment and the displacement of the ring in the radial direction could be equal, for example, TXE(T, 1) = DXI(T, 7) and TYE(T, 1) = free. Furthermore, the displacement of investment and ring in the z-axis direction at the half ring height could also be equal, TYE(T, 5) = DYI(T, 21). The displacement of the ring at step N was therefore calculated.

IIb. Finally, from the calculated displacement of the ring U(N, I), where $\mathrm{U}(N, 2*I-1) = \mathrm{TXE}(N, \mathrm{I})$ and $\mathrm{U}(N, 2*I) = \mathrm{TYE}(N, I)$, the applied external force at each nodal point, F(N, J), can be calculated as $\mathrm{K}(I, J)\mathrm{U}(N, I) = \mathrm{F}(N, J)$, where K($I, J$) is the total

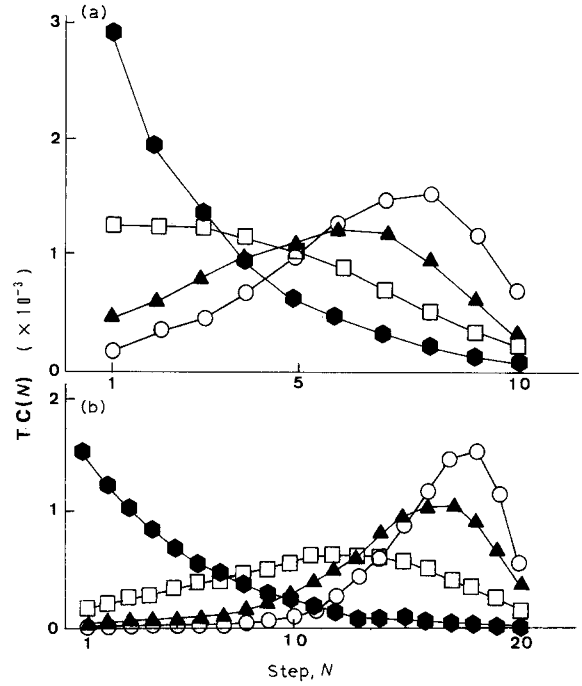

Figure 1 Values of TC(N), the free expansion in step N, at various time division ratios RA: (●) RA = 1.0, (□) RA = 1.2, (▲) RA = 1.4 and (○) RA = 1.6. (a) SN = 10 and (b) SN = 20.

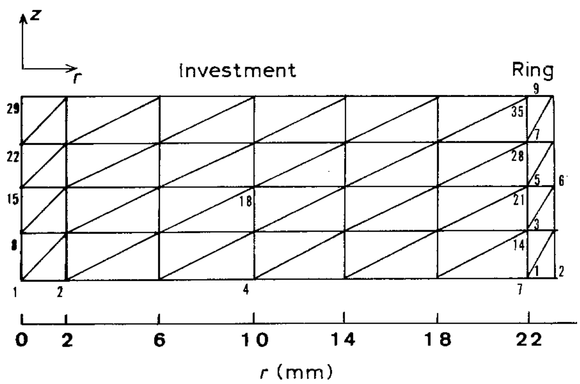

Figure 2 The finite-element model for the setting expansion of an investment within an elastic ring.

stiffness matrix. These values of the external force will be taken into the boundary conditions for the calculation for the investment in the next step, $N + 1$ (Fig. 3). No frictional force between the investment and the ring was considered in this case.

We need to add contact-related boundary conditions between the investment and elastic body to the usual boundary conditions for each analysis. All calculations were made on a personal computer (PC-9801VM, NEC Ltd, Tokyo).

3. Results and discussion

The results of the FEM analysis shown in Fig. 4 represent the expansions of nodal point 21 of the investment. When SN = 10 the results were reasonable for the cases of 0.3 and 1.0 mm thickness. However, the calculated values oscillated in the case of 3.0 mm thickness when SN = 10 for the following reason. Once the pressure applied on the surface of the investment becomes sufficiently large to diminish the investment in step N, the deformation of the ring becomes small in this step N. In the next step $N + 1$,

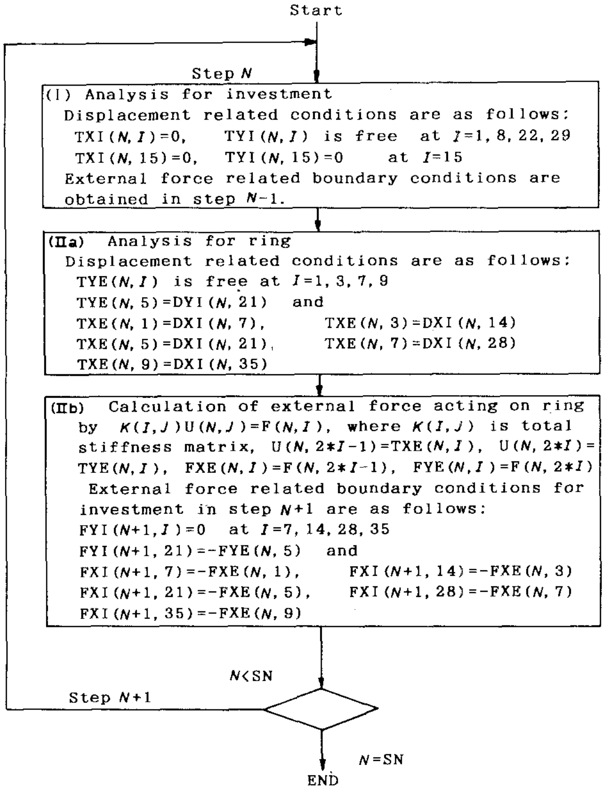

Figure 3 Flowchart of analysis and boundary conditions.

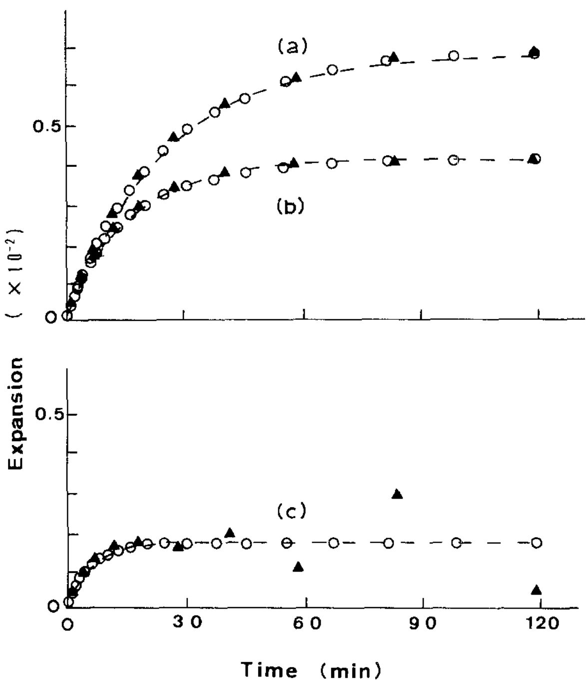

Figure 4 Results of FEM analysis and analytical calculations: (▲) SN = 10 and (○) SN = 20 by the FEM; (– – –) by Equation 6. Thickness: (a) 0.3 mm, (b) 1.0 mm and (c) 3.0 mm.

therefore, the pressure applied on the investment mould could be small. Finally, the expansion in step $N + 1$ is expected to be large and the deformation of the elastic ring should also be large. Then, the ring again applies a large restrictive force on the investment mould in the next step. This oscillation diminished when the total step number SN increased (Fig. 4). In the case of 3.0 mm thickness, the result was

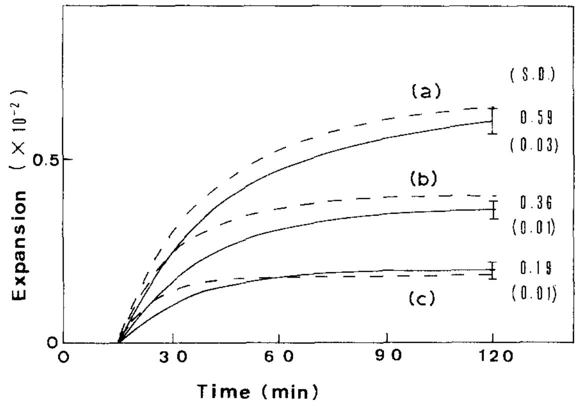

Figure 5 Setting expansion curve at each thickness: (——) measured and (– – –) by the FEM, SN = 20. Thickness (a) 0.3 mm, (b) 1.0 mm and (c) 3.0 mm.

reasonable when the total step number adopted was 20. In Fig. 4 the values calculated for the expansion by Equation 6 are also shown:

$$U = \frac{rLE'}{1 - \nu'}\left[1 - \exp\left(- \frac{(1 - \nu')a_0}{E'L} \times [1 - \exp(- kt)]\right)\right] \quad (6)$$

$$L = \frac{(1 - \nu)A^2 + (1 + \nu)B^2}{E(B^2 - A^2)}$$

where A and B are the inner and outer radii, respectively, and E and ν are the Young's modulus and Poisson's ratio of the ring, respectively. Equation 6 was obtained analytically for the setting expansion within an elastic ring under the same conditions as in [3]. As Fig. 4 shows, the two results obtained by the FEM and Equation 6 agree well with each other.

Fig. 5 shows the measured expansion within the rings reported in [3]. The results from the FEM analysis agreed fairly well with the measured results. In summary, FEM analysis of the setting expansion of a gypsum-bonded investment could be useful, compared with the previous study [3].

Appendix A

The relationships between the stress and strain for the investment are as follows [2]:

$$\frac{\partial \varepsilon_{xx}}{\partial t} = ka_0 \exp(- kt) \times \left(1 + \frac{\sigma_{xx}}{E'} - \frac{\nu'\sigma_{yy}}{E'} - \frac{\nu'\sigma_{zz}}{E'}\right) \quad (A1)$$

$$\frac{\partial \varepsilon_{yy}}{\partial t} = ka_0 \exp(- kt) \times \left(1 + \frac{\sigma_{yy}}{E'} - \frac{\nu'\sigma_{zz}}{E'} - \frac{\nu'\sigma_{xx}}{E'}\right) \quad (A2)$$

$$\frac{\partial \varepsilon_{zz}}{\partial t} = ka_0 \exp(- kt)\left(1 + \frac{\sigma_{zz}}{E'} - \frac{\nu'\sigma_{xx}}{E'} - \frac{\nu'\sigma_{yy}}{E'}\right) \quad (A3)$$

$$\frac{\partial \varepsilon_{xy}}{\partial t} = ka_0 \exp(-kt)(1 + \nu')\tau_{xy}/E' \quad \text{(A4)}$$

$$\frac{\partial \varepsilon_{yz}}{\partial t} = ka_0 \exp(-kt)(1 + \nu')\tau_{yz}/E' \quad \text{(A5)}$$

$$\frac{\partial \varepsilon_{zx}}{\partial t} = ka_0 \exp(-kt)(1 + \nu')\tau_{zx}/E' \quad \text{(A6)}$$

where k, a_0, ν' and E' are constant values obtained from setting expansions under constant loading conditions.

According to the theory of elasticity the relationships between stress and strain are defined as [5]

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E + \alpha T \quad \text{(A7)}$$

$$\varepsilon_{yy} = [\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]/E + \alpha T \quad \text{(A8)}$$

$$\varepsilon_{zz} = [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]/E + \alpha T \quad \text{(A9)}$$

$$\varepsilon_{xy} = (1 + \nu)\tau_{xy}/E \quad \text{(A10)}$$

$$\varepsilon_{yz} = (1 + \nu)\tau_{yz}/E \quad \text{(A11)}$$

$$\varepsilon_{zx} = (1 + \nu)\tau_{zx}/E \quad \text{(A12)}$$

where E is the Young's modulus, ν is the Poisson's ratio and αT is the thermal expansion.

Appendix B: Definitions of terms

SN: total steps number

TD(N): time duration in step N

TIM(N) = $\sum_{K=1}^{N}$ TD(K): time at the end of step N

TC(N): free setting expansion in step N

TXI(N, I): displacement of investment at nodal point I in the radial direction in step N

TYI(N, I): displacement of investment at nodal point I in the z-axis direction in step N

DXI(N, I) = $\sum_{K=1}^{N}$ TXI(K, I): displacement of nodal point I of investment at TIM(N)

DYI(N, I) = $\sum_{K=1}^{N}$ TYI(K, I): displacement of nodal point I of investment at TIM(N)

TXE(N, I): displacement of nodal point I of ring in the radial direction in step N

TYE(N, I): displacement of nodal point I of ring in the z-axis direction in step N

FXI(N, I): applied force on investment at nodal point I in the radial direction in step N

FYI(N, I): applied force on investment at nodal point I in the z-axis direction in step N

FXE(N, I): calculated applied external force at nodal point I in the radial direction in step N

FYE(N, I): calculated applied external force at nodal point I in the z-axis direction in step N

NIE: number of contact points between the investment and elastic body

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